

1953

Electron injection in betatrons and synchrotrons

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ELECTRON INJECTION IN BETATRONS AND SYNCHROTRONS

by

Lawrence W. Von Tersch

**A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY**

Major Subject: Electrical Engineering

Approved:

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1953

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I. INTRODUCTION

A. General Discussion

The general theory of the operation of the betatron has been thoroughly treated during the past few years and great quantities of experimental data on many operating betatrons bear out the major part of this theory. However, there is one aspect of betatron operation which does not seem to have been completely resolved, that is, the problem of electron injection into the betatron.

The problem of injection, as discussed here, is not restricted to "pure" betatrons. There are now in operation many electron synchrotrons which function as betatrons over the initial part of the operating cycle. In general, the same injection problems exist for these synchrotrons and it is a machine of this type which was used for the experimental work pertaining to this thesis.

Electrons are usually supplied to these machines by the use of pulsed electron guns. These guns are placed so that the electrons are started in paths tangential to circles concentric to the normal equilibrium orbit. As the electrons gain energy by betatron action they spiral in toward the equilibrium orbit. At the same time the electrons execute damped radial and vertical oscillations. The exact path of an injected electron is a complicated function of a number of parameters including the field configuration, the radius

of injection and the momentum of injection.

The efficiency of the injection process is extremely low, the amount of charge being captured into stable orbits being but a minute part of the total charge emitted from the gun. Many devious theories have been advanced to account for these seemingly poor results and many ideas have been tried in an attempt to increase the total charge captured.

Most of the improvement techniques have been directed toward the problem of contracting the electron radius at injection, allowing the electrons to miss the injector structure on subsequent revolutions. A related possibility is the attempt to control the form of the radial oscillations and thus miss the injector in this manner.

Another aspect of the injection problem has not been previously treated in such detail and pertains to the correct time of injection and in particular to the effect of electron injection over a finite period of time. The purpose of this thesis is to investigate theoretically the situation existing when injection is attempted for a relatively long period of time, and to determine just when electrons can be accepted from the injector. It then seems quite feasible to determine in what way the time of acceptance may be lengthened, giving as a possible result increased values of captured charge.

B. Review of Literature

There are innumerable papers concerned with the theory, design and operation of various forms of betatrons and synchrotrons. Several of these papers have reference to the injection problem, primarily from the point of view that increased outputs could be obtained if it were assured that injected electrons did not strike the injector structure. Adams¹, Heymann¹⁰, and Jones¹¹ discuss methods for using special equipment to contract the electron orbits in an attempt to do this. Barden^{3,4} and Kerst¹⁴ discuss ways in which the electron beam might provide a tendency toward self-contraction.

The original analysis of the orbits in a betatron was presented by Kerst¹³ and Kerst and Serber¹⁷ while the idea of representing inertial and magnetic forces by an electric potential function seems to be due to Rajchman and Cherry¹⁹. Rajchman and Cherry derive a form of the potential function for non-relativistic injection but do not use it to calculate an acceptance time.

Observations pertaining to the position of the acceptance period have been primarily made by Wideroe²² and Gregg⁹. Wideroe has noted that capture on the leading edge of the injection voltage pulse does not occur for a particular betatron and presents a space-charge theory as an explanation of the phenomenon. Gregg has developed a technique for the approximate measurement of the acceptance time.

This procedure is discussed under "Experimental Procedure." He also notes a failure to capture on the leading edge and gives as an explanation the perturbing effect of electrons captured on the trailing edge of the injection pulse.

C. Injection Time by Momentum-Matching Considerations

The correct time of injection in a betatron or betatron-started synchrotron is commonly calculated by momentum-matching considerations. If the magnetic field B_z can be assumed to be a linear function of time $B_z = \kappa_1 t$ in the region between zero time and injection time t_i , then

$$P_i = e v_i \kappa_1 t_i \quad (1)$$

where P_i is the momentum of the injected electron, r_i is the radius of injection, and e is the electronic charge. The time of injection is

$$t_i = \frac{P_i}{e v_i \kappa_1} \quad (2)$$

If injection takes place in a non-relativistic region the time of injection can be expressed as

$$t_i = \kappa_2 V_i^{1/2} \quad (3)$$

where κ_2 is a constant and V_i is the injection voltage. This

expression is known as the parabolic law of injection.

It seems quite likely that the time of injection as calculated by Eq. (2) or Eq. (3) is correct, but also rather restrictive. The time of injection should not be a single instant but rather an interval of finite width. The width of this interval, or acceptance time, is of importance from the point of view that large values of acceptance time should correspond to large numbers of captured particles.

II. GENERAL SOLUTION OF PROBLEM

A. Calculation of Equivalent Potential Function

All of the forces acting on an injected electron in a betatron can be considered to have been derived from an electric potential. It is convenient to find this equivalent potential function and then study the shape of the potential function for different conditions of injection.

Consider the magnetic fields of the betatron to have circular symmetry or $\frac{\partial}{\partial \theta} = 0$. If injection is performed in the plane containing the final equilibrium orbit only motion in this plane need be considered, letting $\frac{\partial}{\partial z} = 0$. The only component of the magnetic field will be that normal to the plane of the orbit or B_z . In terms of the magnetic vector potential the only component is A_θ .

For an electron of rest mass m_0 the kinetic potential \mathcal{L} can be written as

$$\mathcal{L} = -m_0 c^2 \sqrt{1 - \beta^2} - e r \dot{\theta} A_\theta \quad (4)$$

where

$$\beta^2 = \frac{v^2}{c^2} = \frac{1}{c^2} (\dot{r}^2 + r^2 \dot{\theta}^2 + \dot{z}^2). \quad (5)$$

The general equation of motion is

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{q}_s} \right) - \frac{\partial \mathcal{L}}{\partial q_s} = 0. \quad (6)$$

The equation in θ will be

$$\frac{d}{dt} (m r^2 \dot{\theta} - e r A_{\theta}) = 0 \quad (7)$$

where $m = \frac{m_0}{\sqrt{1-\beta^2}}$.

The equation in r will be

$$e r \dot{\theta} \left(\frac{\partial A_{\theta}}{\partial r} + \frac{A_{\theta}}{r} \right) + m r \dot{\theta}^2 = \frac{d}{dt} (m \dot{r}). \quad (8)$$

Integration of the θ equation with respect to time gives

$$P_{\theta} = m r^2 \dot{\theta} - e r A_{\theta}. \quad (9)$$

The constant P_{θ} could be calculated using known values at injection time.

$$P_{\theta} = P_i r_i - e r_i A_{\theta i} \quad (10)$$

where P_i is the momentum of injection.

A solution for A_{θ} from Eq. (9) gives

$$A_{\theta} = \frac{1}{e r} (m r^2 \dot{\theta} - P_{\theta}). \quad (11)$$

A substitution of Eq. (11) into Eq. (8) gives

$$-er\dot{\theta} \left\{ \frac{\partial A_{\theta}}{\partial r} + \frac{1}{er^2} mr^2 \dot{\theta} + \frac{1}{er^2} (-P_{\theta}) \right\} + mr^2 \dot{\theta}^2 = \frac{d}{dt} (mr) \quad (12)$$

or

$$er\dot{\theta} \left(-\frac{\partial A_{\theta}}{\partial r} + \frac{P_{\theta}}{er^2} \right) = \frac{d}{dt} (mr). \quad (13)$$

An expression for $\dot{\theta}$ can be calculated from Eq. (9) or

$$\dot{\theta} = \frac{1}{mr^2} (P_{\theta} + erA_{\theta}). \quad (14)$$

A substitution of $\dot{\theta}$ into Eq. (13) gives

$$\frac{e}{mr} (P_{\theta} + erA_{\theta}) \left(-\frac{\partial A_{\theta}}{\partial r} + \frac{P_{\theta}}{er^2} \right) = \frac{d}{dt} (mr). \quad (15)$$

Since

$$\frac{\partial}{\partial r} \left(\frac{P_{\theta}}{r} + eA_{\theta} \right) = - \left(\frac{-P_{\theta}}{r^2} + e \frac{\partial A_{\theta}}{\partial r} \right) \quad (16)$$

then

$$-\frac{\partial A_{\theta}}{\partial r} + \frac{P_{\theta}}{er^2} = \frac{1}{e} \frac{\partial}{\partial r} \left(\frac{P_{\theta}}{r} + eA_{\theta} \right). \quad (17)$$

Rewriting Eq. (15) gives

$$-\frac{1}{m} \left\{ \frac{P_{\theta}}{r} + eA_{\theta} \right\} \frac{\partial}{\partial r} \left\{ \frac{P_{\theta}}{r} + eA_{\theta} \right\} = \frac{d}{dt} (mr) \quad (18)$$

or

$$-\frac{1}{2m} \frac{\partial}{\partial r} \left(\frac{P_\theta}{r} + e A_\theta \right)^2 = \frac{d}{dt} (m \dot{r}). \quad (19)$$

Now let it be assumed that the magnetic and inertial forces which do exist could be derived from the space derivative of a radial electric potential V_m . Then

$$-e \frac{d}{dr} (V_m) = \frac{d}{dt} (m \dot{r}) \quad (20)$$

or

$$V_m = \frac{e}{2m} \left(\frac{P_\theta}{er} + A_\theta \right)^2. \quad (21)$$

B. Examination and Significance of Potential Function

The shape and position of the equivalent potential function is of great interest. It will be examined for maxima and minima.

$$\frac{\partial V_m}{\partial r} = \frac{e}{m} \left(\frac{P_\theta}{er} + A_\theta \right) \left(-\frac{P_\theta}{er^2} + \frac{dA_\theta}{dr} \right) = 0. \quad (22)$$

Therefore

$$\frac{P_\theta}{er} + A_\theta = 0 \quad (23)$$

and

$$-\frac{P_\theta}{er^2} + \frac{dA_\theta}{dr} = 0. \quad (24)$$

The information in Eq. (23) is trivial since if it were satisfied, then by Eq. (14) $\dot{\theta} = 0$. However Eq. (24) is of importance; if time were allowed to advance such that $\frac{\partial A_{\theta}}{\partial r}$ became large with respect to $\frac{P_{\theta}}{er^2}$ Eq. (24) would become the two-one betatron flux condition.

$$B_z r^2 \longrightarrow \int_0^r r B_z dr. \quad (25)$$

At any value of time a solution of Eq. (24) would give the instantaneous equilibrium orbit.

The second derivative is obtained to determine whether the extremes indicated by Eq. (24) are maxima or minima.

$$\frac{\partial^2 V_m}{\partial r^2} = \frac{e}{m} \left(r \frac{\partial A_{\theta}}{\partial r} + A_{\theta} \right) \left(\frac{2}{r} \frac{\partial A_{\theta}}{\partial r} + \frac{\partial^2 A_{\theta}}{\partial r^2} \right). \quad (26)$$

If any oscillations in θ are neglected, $r \frac{\partial A_{\theta}}{\partial r} + A_{\theta}$ will be a positive quantity independent of $A_{\theta} = f(r)$. The extreme in the potential function will then be a minimum if

$$\frac{2}{r} \frac{\partial A_{\theta}}{\partial r} + \frac{\partial^2 A_{\theta}}{\partial r^2} > 0. \quad (27)$$

In terms of the magnetic flux density

$$-\frac{r}{B_z} \frac{\partial B_z}{\partial r} < 1. \quad (28)$$

It is quite common to approximate the field distribution in the neighborhood of the equilibrium orbit by

$$B_z = B_0 \left(\frac{r}{r_0} \right)^{-n} \quad (29)$$

The term $\frac{-r}{B_z} \frac{\partial B_z}{\partial r}$ in Eq. (28) is the n of Eq. (29). Kerst and Serber^{13,14} have shown from other considerations that the condition $17n > 0$ is necessary for stable steady-state operation.

To determine whether or not an electron can be injected into a stable orbit the shape and magnitude of the potential function at the time of injection can be considered. There are two general rules. First, the potential function must have a minimum in the appropriate range. Second, the injecting mechanism must lie at a point such that there is no potential maximum greater in magnitude than the injector potential anywhere in the region where capture is expected. If the conditions of Eq. (27) or Eq. (28) are satisfied the second restriction is easily met since there can be no potential maxima in the region under consideration.

If the electrons were injected with an energy such that $P_\theta = 0$, then a minimum in the potential function would most assuredly exist in a position such that electrons could be captured since this is the condition for a "steady-state" orbit. However if P_θ is made sufficiently positive or sufficiently negative the minimum in the potential function may either fail to exist or will exist outside the region of the equilibrium orbit (and possibly outside the region of

the confining donut). It is difficult to show this for the general relationship of Eq. (21) but for specific field distributions the range of P_0 can be calculated.

C. Calculation of Potential Function for $B_z = B_0 \left(\frac{r}{r_0}\right)^{-n}$

Consider the flux distribution in the neighborhood of the equilibrium orbit to be that of Eq. (29). Let the value of A at $r = r_x$ be A_{01} ; therefore A_θ can be written

$$A_\theta = \frac{r_x}{r} A_{01} + \frac{1}{r} \int_{r_x}^r (2-n) A_0 \left(\frac{r}{r_0}\right)^{1-n} dr \quad (30)$$

where

$$A_0 = \frac{B_0 r_0}{2-n}. \quad (31)$$

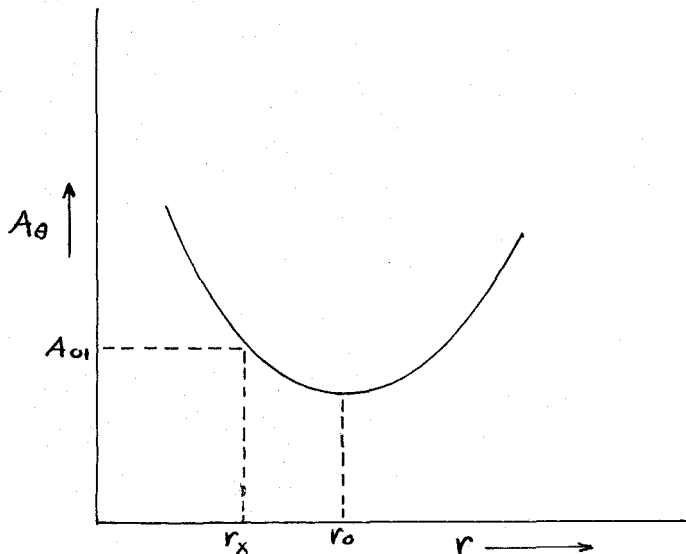


Fig. 1

Assumed Field Configuration

The expression of Eq. (30) can be simplified by taking the steady-state orbit to be at $r = r_0$. From Eq. (24) where $\frac{\partial A_\theta}{\partial r} \gg \frac{P_\theta}{er^2}$,

$$\frac{\partial A_\theta}{\partial r} = -\frac{r_x}{r^2} A_{01} + \frac{1}{r^2} \int_r^{r_x} (z-n) A_0 \left(\frac{r}{r_0}\right)^{1-n} + \frac{(z-n)A_0}{r} \left(\frac{r}{r_0}\right)^{1-n} dr \quad (32)$$

Letting $\frac{\partial A_\theta}{\partial r} = 0$ at $r = r_0$ and solving for A_{01} gives

$$A_{01} = \frac{r_0^2}{r_x} \left\{ \frac{1}{r_0^2} \int_{r_0}^{r_x} (z-n) A_0 \left(\frac{r}{r_0}\right)^{1-n} dr + \frac{(z-n)A_0}{r_0} \right\} \quad (33)$$

A substitution of Eq. (33) into Eq. (30) gives

$$A_\theta = \frac{1}{r} \int_{r_0}^r (z-n) A_0 \left(\frac{r}{r_0}\right)^{1-n} dr + \frac{r_0}{r} (z-n) A_0 \quad (34)$$

$$= A_0 \left\{ \left(\frac{r}{r_0}\right)^{1-n} + \frac{1-n}{\left(\frac{r}{r_0}\right)} \right\} \quad (35)$$

Substituting again into Eq. (24) results in

$$\frac{P_\theta}{er^2} = A_0 \left\{ (1-n) \left(\frac{r}{r_0}\right)^{-n} \left(\frac{1}{r_0}\right) + (r_0)(1-n) \left(-\frac{1}{r^2}\right) \right\} \quad (36)$$

Solving Eq. (36) for r gives

$$\left(\frac{r}{r_0}\right)^{2-n} = 1 + \frac{P_\theta}{er_0 A_0 (1-n)} \quad (37)$$

$$r = r_0 \left\{ 1 + \frac{P_\theta}{er_0 A_0 (1-n)} \right\}^{\frac{1}{2-n}} \quad (38)$$

The instantaneous position of the potential minimum can be calculated from Eq. (38). It must be realized that the potential function under consideration is actually a function of time, that is, both A_0 and A_θ are increasing with time and thus the situation will be altered both as to the possibilities of existence and position of a suitable potential minimum. However if a suitable potential minimum exists at the injection time t_0 , a suitable minimum exists at all times following the injection time and capture is possible. Let

$$A_\theta = A'_\theta(r) f(t). \quad (39)$$

From Eq. (21)

$$V_m = \frac{e}{2m} \left\{ \frac{P_\theta}{e r} + A'_\theta(r) f(t) \right\}^2 \quad (40)$$

$$= \frac{e}{2m} f^2(t) \left\{ \frac{P_\theta}{e r f(t)} + A'_\theta(r) \right\}^2 \quad (41)$$

$$= \frac{e}{2m} f^2(t) \left\{ \frac{P'_\theta}{e r} + A'_\theta(r) \right\}^2. \quad (42)$$

Since $f(t)$ is a monotonically increasing function of time, any P'_θ value which is satisfactory at $t = t_0$ will be satisfactory at any value of time $t > t_0$.

D. Significance of $P_\theta = 0$

Assume that electrons could be introduced into the steady-state orbit under momentum-matching conditions as discussed just before Eq. (1). This should be the optimum situation as far as capture is concerned. At the steady-state equilibrium orbit,

$$2\pi \int_0^r r B_z dr = 2\pi r_o^2 B_z(r_o). \quad (43)$$

To match momentum

$$\frac{m (r_o \dot{\theta})^2}{r_o} = e B_z(r_o) r_o \dot{\theta} \quad (44)$$

or

$$B_z(r_o) = \frac{m \dot{\theta}}{e}. \quad (45)$$

Then

$$\int_0^{r_o} r B_z dr = \frac{r_o^2 m \dot{\theta}}{e}. \quad (46)$$

The conditions of Eq. (46) would make $P_\theta = 0$. In addition it would follow that for outside injection it would be a move in the correct direction to increase the momentum with an increase in injection radius, assuming all other parameters are kept unchanged.

III. USE OF POTENTIAL FUNCTION TO DETERMINE ACCEPTANCE VALUES

A. Single-Instant Injection

A problem analogous to the acceptance-time problem is that where injection is performed at $t = t_0$ for an infinitesimally small length of time. A range of injection voltage is available; the problem being to determine the range of injection voltage allowing capture as a function of injection voltage. The change in mass over the range of momenta to be encountered will be assumed small.

From Eq. (10)

$$P_\theta = P_c r_i - e r_i A \phi_i. \quad (47)$$

Consider the potential function at $t = t_0$. There could be determined an allowable range of P_θ , ΔP_θ , to give suitable potential functions which would allow capture. From Eq. (47) the allowable range in injected momenta ΔP_i is proportional only to ΔP_θ , that is, the range of acceptable momenta is independent of the position of the range. If the conditions of Eq. (27) or Eq. (28) are satisfied then the change in mass with injection momenta is not important since no potential maximum will exist in the critical region.

In terms of the injection voltage the momentum is

$$P_c = \frac{1}{c} \sqrt{(V_i e)(V_i e + 2 m a c^2)} \quad (48)$$

or

$$V_i = \frac{m_0 c^2}{e} \left\{ \sqrt{1 + \left(\frac{P_i}{m_0 c}\right)^2} - 1 \right\}. \quad (49)$$

$$\frac{\partial V_i}{\partial P_i} = \frac{P_i}{m_0 e \sqrt{1 + \left(\frac{P_i}{m_0 c}\right)^2}}. \quad (50)$$

For small values of ΔP_i

$$\Delta V_i = \frac{P_i}{m_0 e \sqrt{1 + \left(\frac{P_i}{m_0 c}\right)^2}} \Delta P_i \quad (51)$$

or

$$\Delta V_i = \frac{c \sqrt{(V_i e)(V_i e + 2 m_0 c^2)}}{e \sqrt{(m_0 c^2)^2 + (V_i e)(V_i e + 2 m_0 c^2)}} \Delta P_i. \quad (52)$$

The term ΔP_i can be used as a constant, allowing the range of allowable injection voltage to be calculated for any given injection voltage.

B. Constant-Momentum Injection

A similar but more useful problem is that of constant voltage or constant momentum injection. In this case the time interval of successful capture, or acceptance time, will be calculated. The potential function can be written

$$V_m = \frac{e}{2m} f^2(t) \left\{ A'_0(r) + \frac{P_i V_i}{e r f(t)} - \frac{r_i A_{0i}}{r f(t)} \right\}^2. \quad (53)$$

The mass m corresponds to the appropriate injection momentum.

The potential function is of primary interest at injection, that is, for $f(t) = f(t_0)$. Then

$$V_{mi} = \frac{e}{2m} f^2(t_0) \left\{ A'_\theta(r) + \frac{\frac{P_i r_i}{f(t_0)} - e r A'_\theta(r_i)}{e r} \right\}^2 \quad (54)$$

$$= \frac{e}{2m} f^2(t_0) \left\{ A'_\theta(r) + \frac{P''_\theta}{e r} \right\}^2 \quad (55)$$

where

$$P''_\theta = \frac{P_i r_i}{f(t_0)} - e r_i A'_\theta(r_i). \quad (56)$$

The time function in front of the squared bracket will not alter the shape of the potential function but only the overall magnitude.

Thus there is again one "sure" value of P''_θ where V_m will have a suitable minimum. There is also a value of P''_θ which will be too great and one which will be too small, giving an allowable range in P''_θ . This range in P''_θ is

$$\Delta P''_\theta = (P''_\theta)_{max.} - (P''_\theta)_{min.} \quad (57)$$

$$= \frac{P_i r_i}{f(t_1)} - \frac{P_i r_i}{f(t_2)} \quad (58)$$

or

$$f(t_2) - f(t_1) = \frac{f(t_1) f(t_2)}{P_i r_i} \Delta P''_\theta. \quad (59)$$

The term $f(t_1) f(t_2)$ may be approximated by the square of the time function where $P_{\theta}'' = 0$,

$$\{f(t_0)\}^2 \approx f(t_1) f(t_2). \quad (60)$$

From Eq. (56)

$$\{f(t_0)\}^2 = \left\{ \frac{P_i}{e A_{\theta}'(r_i)} \right\}^2 = \frac{P_i^2}{e^2 \{A_{\theta}'(r_i)\}^2} \quad (61)$$

or

$$f(t_2) - f(t_1) = \frac{\{f(t_0)\}^2}{P_i r_i} \Delta P_{\theta}'' \quad (62)$$

$$= \frac{P_i}{e^2 r_i \{A_{\theta}'(r_i)\}^2} \Delta P_{\theta}'' \quad (63)$$

In most cases injection takes place so close to the time zero that the time function can be represented as in Eq. (1), that is,

$f(t) = \kappa_1 t$. Then

$$\Delta t = \frac{P_i}{\kappa_1 e^2 r_i \{A_{\theta}'(r_i)\}^2} \Delta P_{\theta}'' \quad (64)$$

A further clarification of $\Delta P_{\theta}''$ is possibly needed. From Eq. (55)

$\Delta P_{\theta}''$ is dependent only upon $A_{\theta}'(r)$. Changing the field configuration is the only way to change the allowable value of $\Delta P_{\theta}''$.

Writing $\Delta P_{\theta}'' = f_1(r)$,

$$\Delta t = \frac{P_i \cdot f_1(r)}{\kappa_1 e^2 r_i \{A_{\theta}'(r_i)\}^2} \quad (65)$$

Several results seem evident in Eq. (65). First the acceptance time varies inversely as the rate of change of magnetic field. It seems logical to believe that the injection process should be simpler for slowly changing magnetic fields as compared to rapidly changing systems.

Second, $\Delta \tau$ is directly proportional to the injection momenta. It is hard to realize a simple physical explanation for this relationship but there is a considerable amount of experimental data which might be construed as affirmation of this relationship. Most betatrons and synchrotrons exhibit increased gamma-ray output for increases in injection momenta.

The variation of the acceptance time with the radius of injection is also difficult to show in the general case but can be seen for the situation specified in Eq. (29). Since Eq. (35) has a minimum at $r = r_0$, $\{A\delta(r)\}^2$ is increasing for increasing values of r_1 . Thus the acceptance time would increase to a maximum as the injection radius is moved toward the steady-state equilibrium radius. However it must be realized that other factors may actually cause the total number of electrons captured to be decreased instead of increased by virtue of such a move.

C. Pulsed Injection Systems

A second problem concerning the calculation of acceptance

time comes from the use of an injection pulse similar to that shown in Fig. 2.

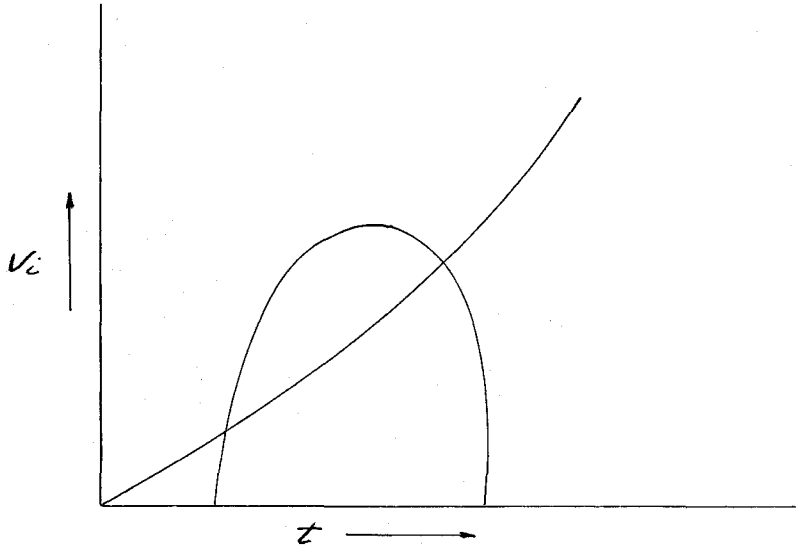


Fig. 2 Injection System Showing Two Acceptance Periods

The parabolic curve is obtained by substituting Eq. (2) into Eq. (49).

This gives

$$v_i = \frac{m_0 c^2}{e} \left\{ \sqrt{1 + \left(\frac{v_i \kappa_i e t_i}{m_0 c} \right)^2} - 1 \right\}. \quad (66)$$

The parabola as shown in Fig. 2 and Eq. (66) is for the particular case as discussed before, that is, the magnetic field in this region can be represented as a linear time function $f(t) = \kappa_i t$.

However for this case and many similar ones there are two possibilities for momentum matching, giving two different time intervals in which particles may be captured. Considering P_1 as a time function

and rewriting Eq. (56) gives

$$P_{\theta}'' = \frac{P_i(t) v_i}{f(t)} - e v_i A_{\theta}'(v_i). \quad (67)$$

The change in mass with injection momentum will not alter the situation since no maxima will be presumed to exist in the critical region.

A generalization of the time functions as shown in Eq. (67) makes it exceedingly difficult to make any statements as to the effect of injection voltage waveshapes on acceptance time beyond the obvious observation that there should be a tendency toward momentum matching on the positive slope. Thus any calculations should show the possibility of an increased acceptance time where $\frac{\partial V_i}{\partial t} > 0$.

In an attempt to gain some idea what form the results might take, a crude approximation to the injection momentum waveform is shown in Fig. 3.

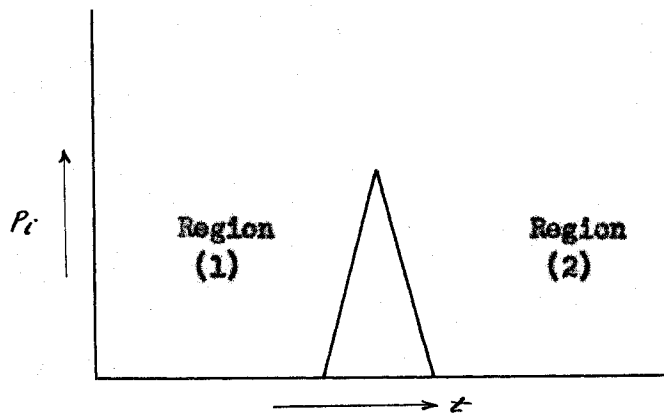


Fig. 3 Approximation to Injection Waveform

In region (1) where $\frac{\partial P_i}{\partial t} > 0$ let $P_i(t) = \alpha t - \gamma$ and in region (2) where $\frac{\partial P_i}{\partial t} < 0$, $P_i(t) = -\alpha t + \rho$. The quantities α , ρ , and γ are all positive real numbers. The following two equations can then be written from Eq. (67).

In region (1)

$$P_{\theta}'' = \frac{\alpha r_i}{\kappa_i} - \frac{\gamma r_i}{\kappa_i t} - e r_i A_{\theta}'(r_i). \quad (68)$$

In region (2)

$$P_{\theta}'' = -\frac{\alpha r_i}{\kappa_i} + \frac{\rho r_i}{\kappa_i t} - e r_i A_{\theta}'(r_i). \quad (69)$$

Somewhere in region (1) there is a time where $P_{\theta}'' = 0$. This time is

$$t_{01} = \frac{\gamma}{\kappa_i \left\{ \frac{\alpha}{\kappa_i} - e A_{\theta}'(r_i) \right\}}. \quad (70)$$

The minimum value of P_{θ}'' occurs at a lower value of t than does the maximum. Thus for region (1),

$$\Delta P_{\theta}'' = \frac{\gamma r_i}{\kappa_i} \left\{ \frac{t_2 - t_1}{t_1 t_2} \right\} \quad (71)$$

or

$$\Delta t = t_2 - t_1 = \frac{\kappa_i t_1 t_2}{\gamma r_i} \Delta P_{\theta}'' \quad (72)$$

Writing $t_1, t_2 \approx t_{01}^2$ and substituting from Eq. (70) gives

$$\Delta t = \frac{\gamma}{r_i \kappa_i \left\{ \frac{\alpha}{\kappa_i} - e A_{\theta}'(r_i) \right\}^2} \Delta P_{\theta}'' \quad (73)$$

The time in region (2) for which $P_{\theta}'' = 0$ is

$$t_{02} = \frac{P}{\pi_1 \left\{ \frac{\alpha}{\pi_1} + e A_{\theta}'(v_i) \right\}} \quad (74)$$

In region (2) the minimum value of P_{θ}'' occurs for the larger critical value of t . Then for region (2),

$$\Delta P_{\theta}'' = \frac{P v_i}{\pi_1} \left\{ \frac{t_2 - t_1}{t_1 t_2} \right\}. \quad (75)$$

Again writing $t_1, t_2 \approx t_{02}^2$ and solving for $\Delta t = t_2 - t_1$ gives

$$\Delta t = \frac{P}{\pi_1 v_i \left\{ \frac{\alpha}{\pi_1} + e A_{\theta}'(v_i) \right\}^2} \Delta P_{\theta}'' \quad (76)$$

As a mathematical check let $\alpha \rightarrow 0$ and $P \rightarrow P_i$, a constant value independent of time. Rewriting Eq. (76) then gives

$$\Delta t = \frac{P_i}{\pi_1 v_i e^2 \left\{ A_{\theta}'(v_i) \right\}^2} \Delta P_{\theta}'' \quad (77)$$

This equation is identical with that of Eq. (64).

The expressions for the acceptance times on the leading and trailing edges of the injection pulse are very similar except for the negative sign in the denominator of Eq. (73). It would seem possible to obtain a form of momentum matching and larger acceptance times by letting

$$A_{\theta}'(v_i) = \frac{\alpha}{\pi_1 e} \quad (78)$$

For other than linear time functions whose quotient is not so easily expressible there should still be a relationship giving a maximum acceptance time in region (1).

About the only thing that can be done mathematically toward improving the acceptance time on the trailing edge of the injection pulse is to use a shallow trailing slope. However this may not necessarily be a very practical solution.

Unfortunately the apparent theoretical advantages of capture in the region of $\frac{\partial P_i}{\partial t} > 0$ as obtained from the acceptance-time point of view do not seem to be realized in many cases. Both Wideroe²² and Gregg⁹ have observed a failure to capture electrons in their respective betatrons where $\frac{\partial P_i}{\partial t} > 0$. The most logical reason for this, given by Gregg, is that the magnetic disturbance created by the capture of electrons on the trailing edge of the injection pulse, perturbs the orbits of those previously captured by an amount sufficient to cause them to hit the walls of the confining donut. The only electrons then to fall into stable orbits are the relatively small quantity captured on the trailing edge.

It has also been observed that pulsing the magnetic field at injection time has had the effect of increasing the yield of many betatrons. Many different and dubious explanations have been advanced for this improvement and by reason of the preceding equations one more may possibly be added to the list. If the magnetic field were perturbed in such a way as to cancel out the effect of the electrons injected on the trailing edge, then the electrons captured on the rising slope could be retained. Whether or not the second

bunch of electrons is preserved seems of little importance since the time interval of Eq. (73) may be many times that of Eq. (76).

IV. NUMERICAL CALCULATIONS FOR ISC SYNCHROTRON

A. Determination of Potential Function

The potential function for a typical machine, the Iowa State College synchrotron, will be approximated¹. The following assumptions will be made.

1. The field configuration follows the form of Eqs. (29) and (35) for the region $0.2540 < r < 0.3302$ meters. (11-13 inches)
2. The value of n in Eqs. (29) and (35) is 0.67.
3. Experimentally it is known that the transition from betatron to synchrotron operation can be made to take place at a time 200 microseconds following zero time and at an energy of approximately 3.2×10^{-13} joules. (2 mev.)
4. The increase of the magnetic field is linear with respect to time during the betatron period of operation.
5. Injection can take place in the neighborhood of 15 microseconds after zero time.
6. The radius of injection will be used as 0.3302 meters. (13 inches)

First, the most desirable potential function will be determined; that existing if injection is performed such that $\theta = 0$.

For a discussion of this machine see Elder⁷ and Pollock¹⁸.

From Eq. (10)

$$P_{\theta} = P_i r_i - e v_i A_{\theta i} \quad (79)$$

or the injection momenta will be

$$P_i = e A_{\theta i}. \quad (80)$$

The magnitude of $A_{\theta i}$ can be approximated from the known time and energy values. An electron energy of 3.2×10^{-13} joules corresponds to a momentum of $P = 13.11 \times 10^{-22}$ kg. meter/sec., or at the equilibrium orbit of $r = 0.2921$ meters the magnetic flux density is

$$\begin{aligned} B(r_0, 200 \mu\text{sec}) &= \frac{P}{e r_0} = \frac{(13.11)(10)^{-22}}{(0.2921)(1.6)(10)^{-19}} \\ &= 280 \times 10^{-4} \frac{\text{webers}}{\text{meter}^2}. \end{aligned} \quad (81)$$

Assuming the rate of rise of the magnetic field to be uniform and injection to take place at $t = 15$ microseconds gives

$$B(r_0, t_0) = 21 \times 10^{-4} \frac{\text{webers}}{\text{meter}^2}. \quad (82)$$

Also

$$A_{\theta}(r_0, t_0) = 6.137 \times 10^{-4} \frac{\text{webers}}{\text{meter}}. \quad (83)$$

From Eq. (35) at $r_1 = 0.3302$ meters, $A_{\theta i}$ can be calculated as

$$A_{\theta i} = A_{\theta}(r_i, t_0) \left\{ \left(\frac{r_i}{r_0} \right)^{1-n} + \frac{1-n}{\left(\frac{r_i}{r_0} \right)} \right\}$$

$$= 8.1778 \times 10^{-4} \frac{\text{webers}}{\text{meter}}.$$
(84)

The required injection momentum is $P_i = eA_{\theta i}$, and the corresponding injection voltage is

$$V_i = \frac{m_0 c^2}{e} \left\{ \sqrt{1 + \left(\frac{eA_{\theta i}}{m_0 c} \right)^2} - 1 \right\}$$

$$= 55,447 \text{ volts}.$$
(85)

This is the value of injection voltage needed to make $P_{\theta} = 0$ if injection takes place at $t = 15$ microseconds at a radius of 13 inches.

From Eq. (21) where $P_{\theta} = 0$,

$$V_m = \frac{e}{2m} (A_{\theta})^2 = \frac{eA_{\theta}^2}{2m} \left\{ \left(\frac{r}{r_0} \right)^{1-n} + \frac{1-n}{\left(\frac{r}{r_0} \right)} \right\}^2.$$
(86)

At an injection voltage of 55,447 volts, $m = 1.109 m_0$ and

$$\frac{eA_{\theta}^2}{2m} = 29,856 \text{ volts}.$$
(87)

Table 1

Calculated Potential Function for ISG Synchrotron, $P_0 = 0$

r (meters)	r (inches)	V_m (volts)
0.2540	10.00	53,163
0.2604	10.25	53,002
0.2667	10.50	52,960
0.2731	10.75	52,894
0.2794	11.00	52,849
0.2858	11.25	52,821
0.2921	11.50	52,798
0.2985	11.75	52,821
0.3048	12.00	52,844
0.3115	12.25	52,890
0.3175	12.50	52,938
0.3239	12.75	52,994
0.3302	13.00	53,068

For the given value of n , V_m can be calculated as a function of the radius. The results of this calculation are shown in Table 1 and Fig. 4.

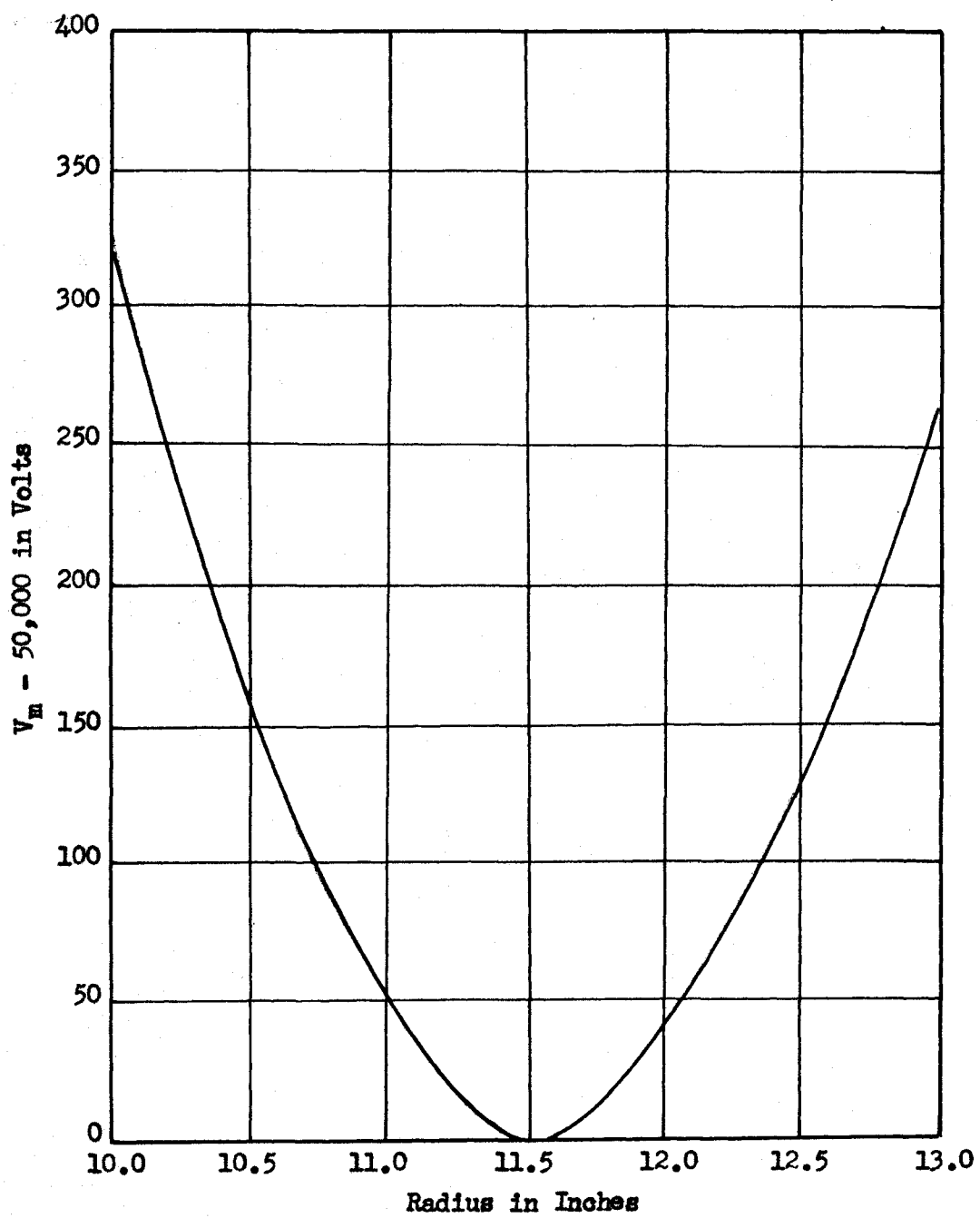


Fig. 4 Typical Potential Function

B. Determination of Acceptance Time

A typical potential curve has been calculated. However the possibly naive assumption for the shape of the field configuration gives difficulty in calculating acceptance times. As long as the form of Eq. (35) is used only the position of the potential minima can be used to approximate the acceptance time since a minimum of suitable shape (but not necessarily suitable position) will always exist.

Assume an electron can be captured into a stable orbit if the minimum in the potential function exists anywhere for $0.2540 < r < 0.3302$ meters. The injection radius will be the same as that of the previous example, that is 0.3302 meters. It is not plausible, with the injector structure used, to consider any electrons to be captured if the potential minimum is outside the injection radius. Similarly the assumed minimum radius is sufficiently close to the inside radius of the donut to cause electrons approaching a minimum inside this radius to be lost. From Eq. (37)

$$\frac{P_0}{e r_0 A_0 (1-n)} = \left(\frac{r}{r_0} \right)^{1-n} - 1 \quad (88)$$

or

$$\frac{e r_0 (1-n)}{r_i P_i} \left\{ \left(\frac{r}{r_0} \right)^{2-n} - 1 \right\} = \frac{1 - \frac{e A_0 \epsilon_i}{P_i}}{A_0} \quad (89)$$

Writing A_{ei} and A_o as linear time functions and then solving for t gives

$$t = \frac{t_o}{1 + \frac{(r_o)(1-n) \left\{ \left(\frac{r_o}{r_i} \right)^{2-n} - 1 \right\}}{(r_i) \left\{ \left(\frac{r_i}{r_o} \right)^{1-n} + \frac{1-n}{\left(\frac{r_i}{r_o} \right)} \right\}}} \quad (90)$$

For a selected value of t_o there is an injection momentum such that $P_e = 0$. If this injection momentum is maintained for other values of t , the time at which the minimum in the potential function is at a given value of r can be calculated from Eq. (90).

Substituting the appropriate values into Eq. (90) gives

$$t = \frac{t_o}{1 + 0.4435 \left\{ \left(\frac{r_o}{r_i} \right)^{2-n} - 1 \right\}} \quad (91)$$

Numerical values are shown in Table 2 and are plotted in Fig. 5.

In the previous example an injection voltage of 55,447 volts gave $P_e = 0$ for an injection time of 15 microseconds. Calculating t as a function of r for this case gives an acceptance time of 2.39 microseconds. The acceptance time as a function of t_o is

$$\Delta t = 0.1595 t_o. \quad (92)$$

Also

$$P_i = 8.723 \times 10^{-24} t_o. \quad (93)$$

where t_o is in microseconds and P_i is in kg. meter/sec.

These results are tabulated in Table 2 and plotted in Fig. 6.

Table 2

Calculated Acceptance Times for ISC Synchrotron

P_1 (kg.-meter/sec.)	P_1 (gauss-cm.)	V_1 (volts)	t_0 (microsec.)	t (microsec.)
12.212 x 10 ⁻²³	763.25	48,841	14.0	2.233
12.387 "	774.18	50,173	14.2	2.265
12.561 "	785.06	51,557	14.4	2.297
12.736 "	796.00	52,941	14.6	2.329
12.910 "	806.97	54,325	14.8	2.360
13.048 "	817.78	55,447	15.0	2.392
13.259 "	828.68	57,143	15.2	2.424
13.434 "	839.63	58,578	15.4	2.456
13.608 "	850.50	60,113	15.6	2.488
13.782 "	861.37	61,500	15.8	2.520
13.957 "	872.31	62,986	16.0	2.552

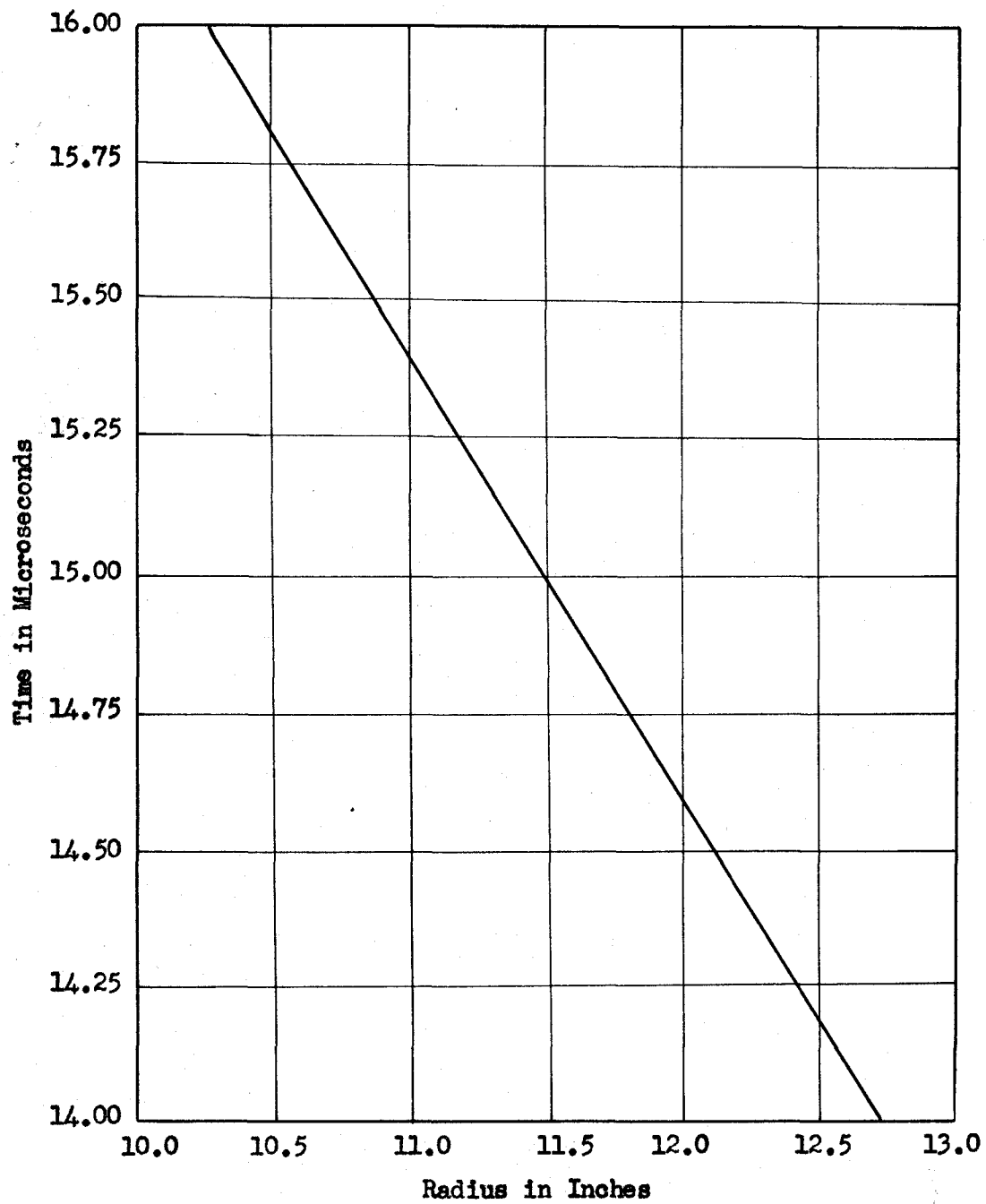


Fig. 5 Position of Voltage Minimum As Function of Time

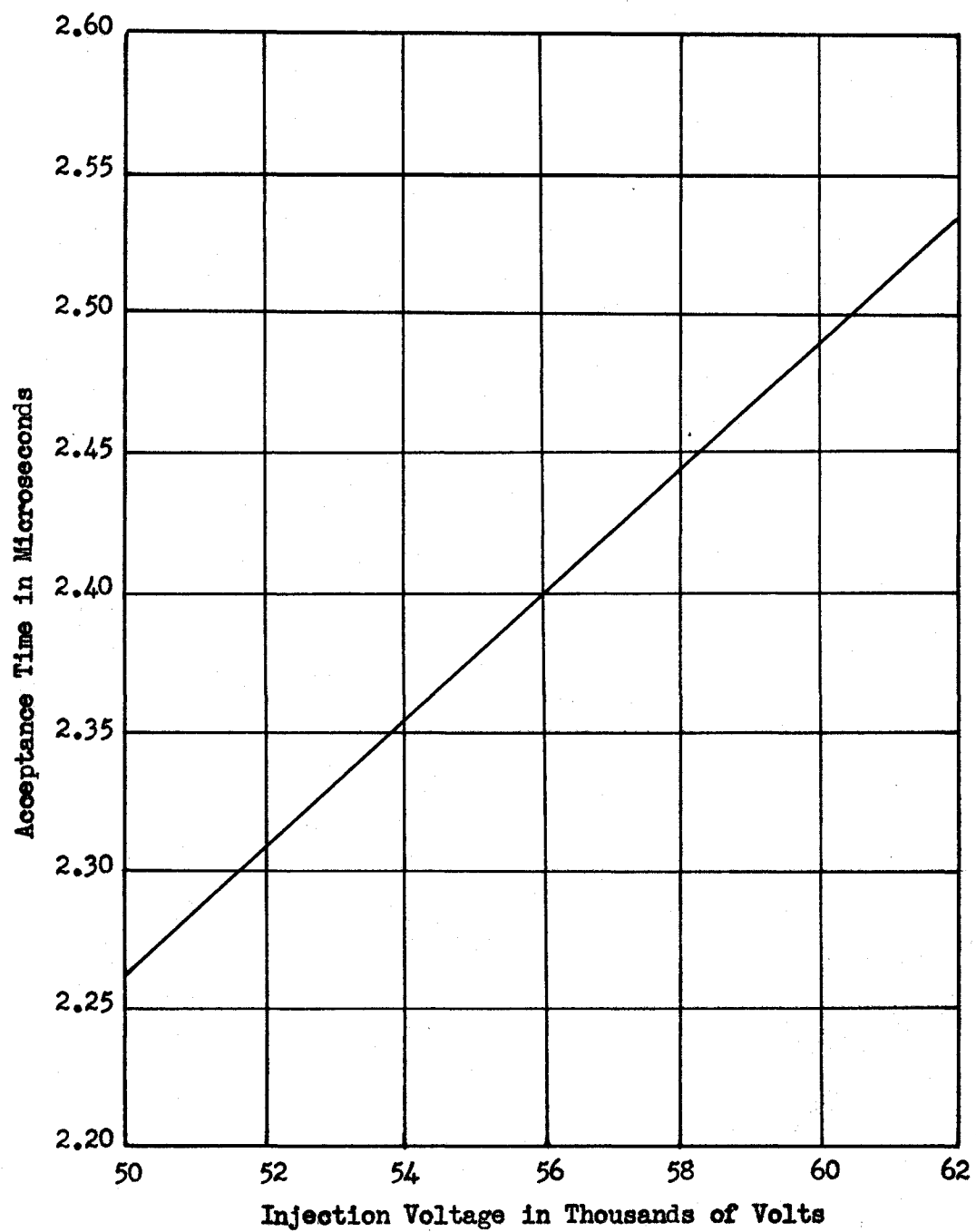


Fig. 6 Acceptance Time As A Function of Injection Voltage

V. EXPERIMENTAL PROCEDURE

A. Theory of Experimental Measurements

Some rather simple experiments may be performed in an attempt to measure the location and magnitude of the acceptance time. A procedure for this type of measurement has been suggested by Gregg⁹.

Assume the acceptance time to be small with respect to the width of the injection voltage pulse. Also consider the electron beam to be most sensitive to perturbations during the period of acceptance. A perturbing magnetic pulse can then be used to probe around during the suspected acceptance time with the correct period being indicated by a drop in gamma-ray output.

Let the acceptance time be sharply defined and the perturbing pulse be square as shown in Fig. 7. The perturbing pulse should be equal to or less than the acceptance time in width. If the machine is adjusted for a given output and the perturbing pulse is moved in from zero time, the output will remain constant until the trailing edge of the perturbing pulse reaches the front edge of the acceptance period. The output will then drop until the trailing edge of the perturbing pulse passes the end of the acceptance time. Beyond this point the output should again rise to its original value since the machine fields are becoming large with respect to the disturbing fields. The acceptance time would then be that over which a

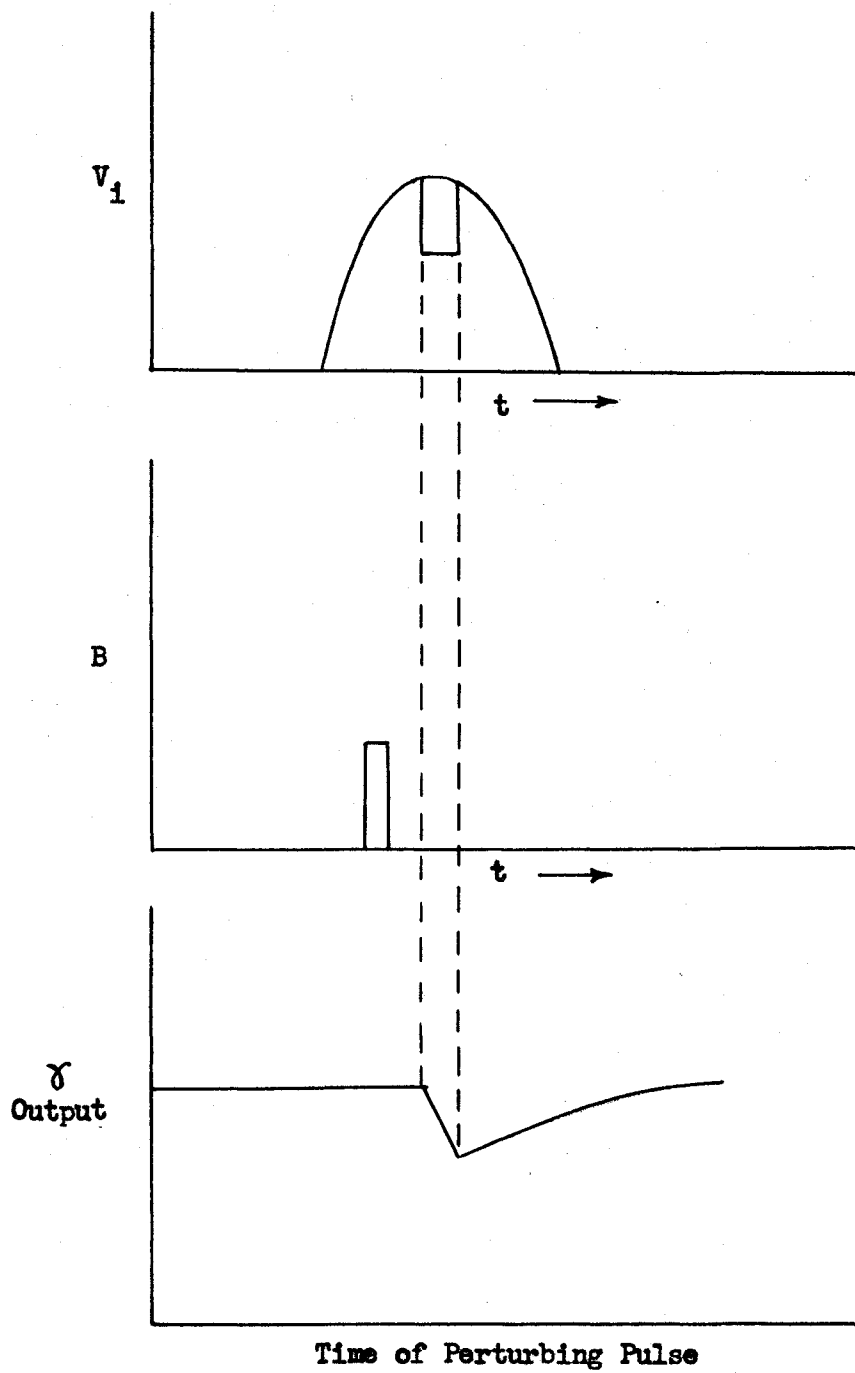


Fig. 7 Measurement of Acceptance Time With Perturbing Pulse

negative slope is obtained in the sketch of Fig. 7c.

B. Arrangement of Equipment

To provide the perturbing fields a system of three similar coils as shown in Fig. 8 was constructed. It was not intended to use the coils simultaneously; the only reason for having more than one coil was to investigate the effect of azimuthal position. However one coil became shorted in the assembly process and was not repaired in time to be used for the tests. Each coil consists of two turns of #30 wire. The coils are centered on a radius of $11 \frac{3}{8}$ inches with the exception of coil C which is centered on a radius of $12 \frac{3}{8}$ inches. The inner edges of the coils are arcs with radii of $10 \frac{3}{8}$ inches and the outer edges are arcs with radii of $12 \frac{3}{8}$ inches.

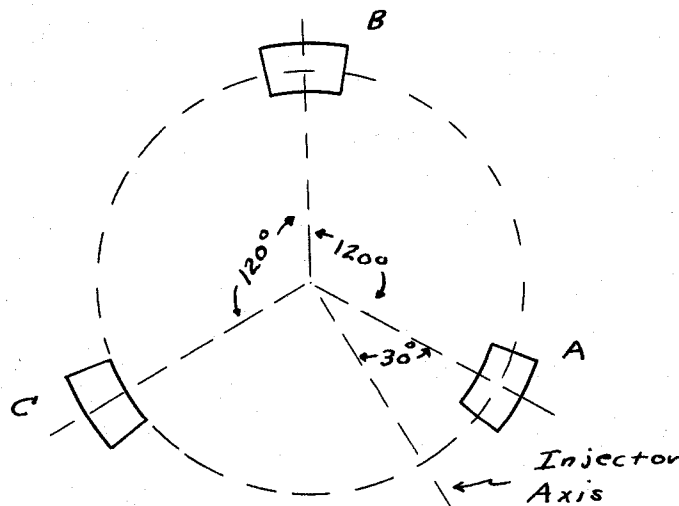


Fig. 8 Perturbing Coil System

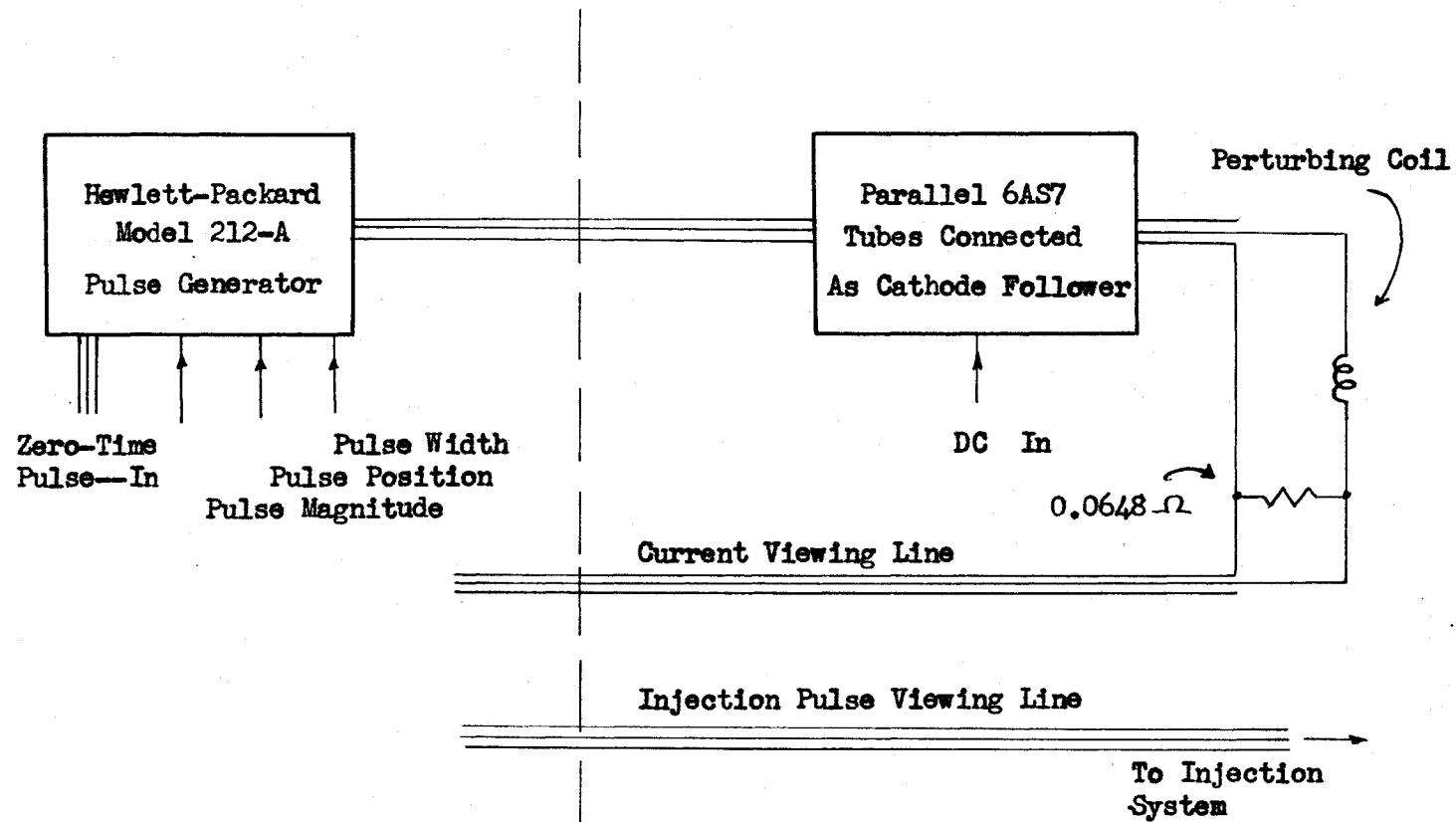


Fig. 9 Perturbing Pulse Source

The area of each coil is 7.8 square inches. The coils were placed between sheets of wrapping paper heavily coated with varnish and glyptal to keep them in place. Coil C was placed farther out than were coils A and B only to avoid the lead-in connections to several other coils which were constructed in the same assembly. These additional coils were designed as flux-measuring coils and have no direct relationship to the acceptance-time problem.

The perturbing coils were driven by a pulse source as indicated in Fig. 9. A zero-time pulse is available at the machine operating position and this is used to synchronize the Hewlett-Packard pulse generator. The current pulse magnitude, position, and width controls are then available at this point. The position of the true time zero is not easily obtained since the zero-time pulse is not conveniently short or symmetrical in shape. The output of the Hewlett-Packard pulse generator is fed to a current-pulse amplifier adjacent to the perturbing coils. The schematic diagram of this circuit is shown in Fig. 10. The amplifier circuit is connected to the perturbing coil by 6 1/2 feet of RG-8U coaxial line. A resistor of 0.0648 ohms is placed in series with the coil to provide a means of viewing the current wave shape. There is nothing particular about this value, 0.0648 ohms being obtained by arranging seventeen 1.1 ohm resistors in parallel to obtain a non-inductive shunt. In as much as the length of line bringing back the current pulse to be viewed is approximately the same length as that for the injection

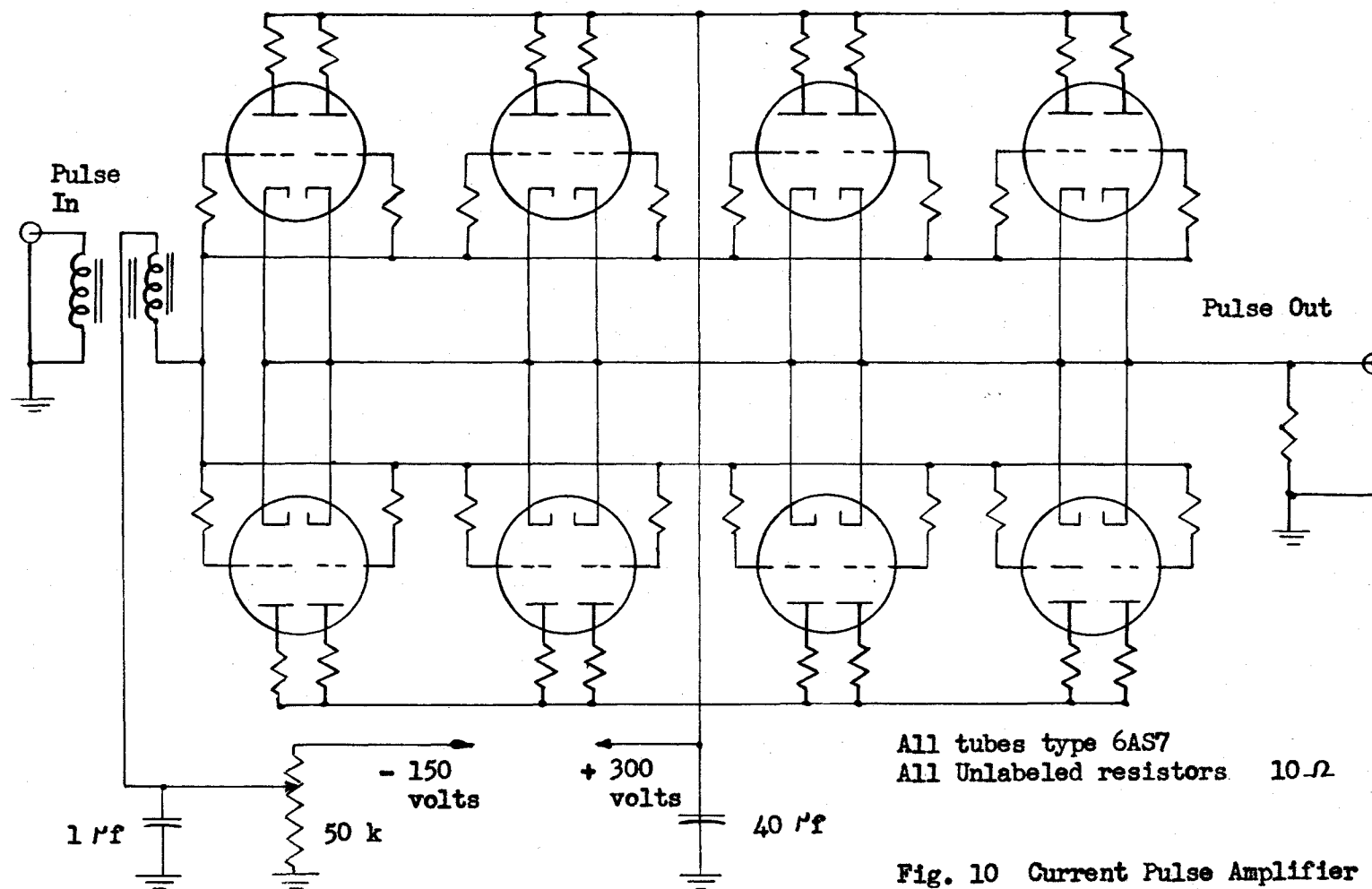


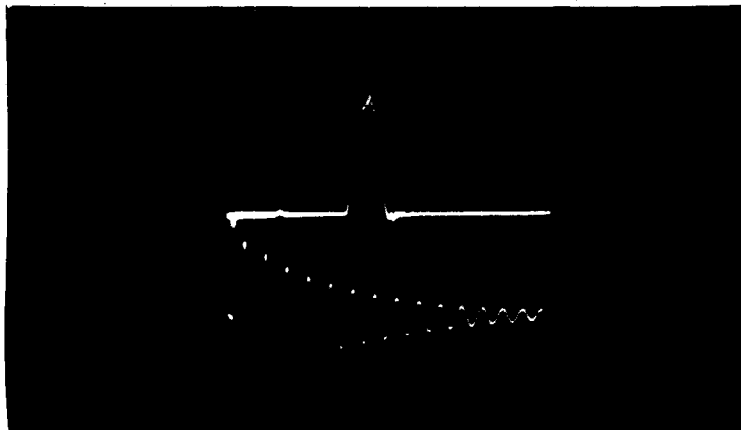
Fig. 10 Current Pulse Amplifier

voltage pulse, the two pulses will appear in the correct relationship.

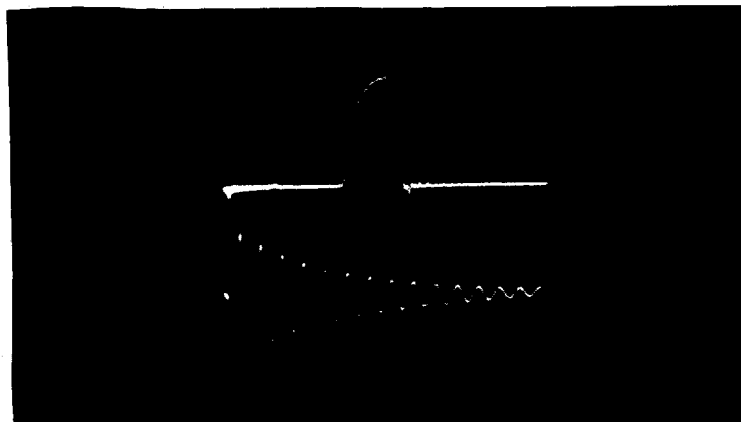
Actually the current pulses are not as square as would be desired. Figs. 11, 12 and 13 show some typical waveforms, that is, waveshapes for pulses 1.8 microseconds to 9 microseconds in length. In Fig. 13a is shown the voltage across the perturbing coil with a 3 microsecond current passing through the coil. In all probability this is not a true picture due to the excessive loading of the oscilloscope. In Fig. 13b is shown the relationship between the voltage and the current for the 1.8 microsecond case. The timing signal has a period of 1 microsecond in all photographs with the exception of Fig. 12b; here the period is 10 microseconds. The magnitude of the current does not appreciably affect the waveshapes, pulse amplitudes from 0 to 30 amperes being obtainable. In addition no effect on current shape or magnitude was noted when the coils were placed in position on top of the accelerating donut in proximity to the iron pole faces of the machine.

C. Experimental Procedure

An attempt was first made to determine the location of the acceptance-time period. The machine was placed in operation under the following conditions.

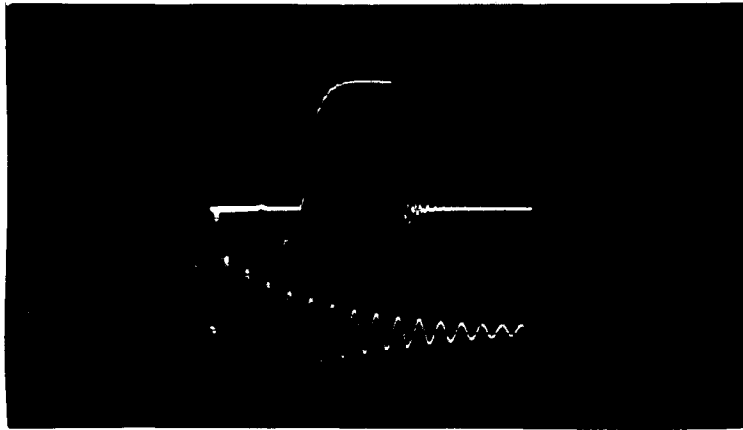


(a) 1.8 Microsecond Current Pulse

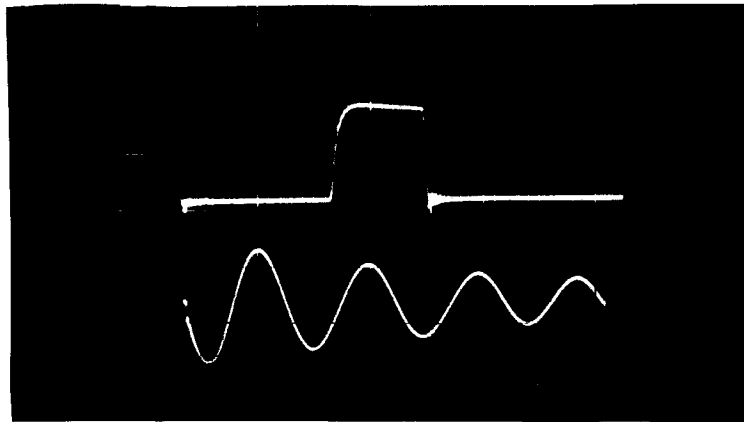


(b) 3 Microsecond Current Pulse

Fig. 11 Current Pulse Waveforms

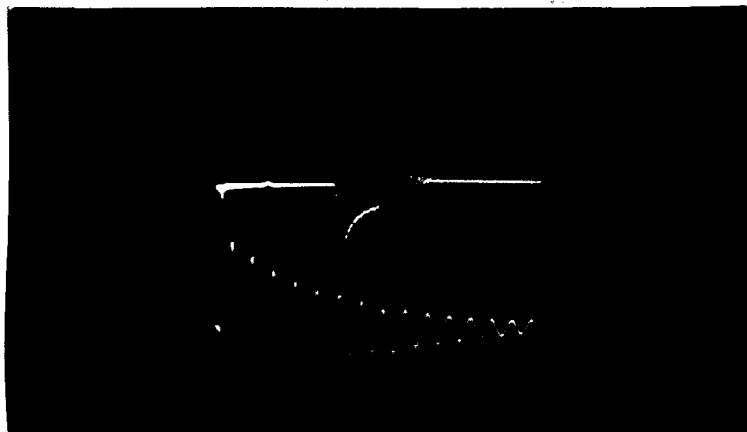


(a) Five Microsecond Current Pulse

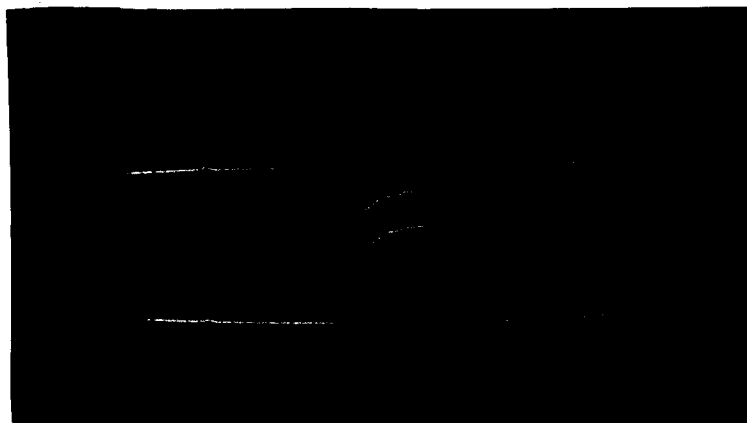


(b) 9 Microsecond Current Pulse

Fig. 12 Current Pulse Waveforms



(a) 3 Microsecond Voltage Pulse



(b) 1.8 Microsecond Current and Voltage Pulses

Fig. 13 Voltage and Current Waveforms

Peak Injection Voltage --- 49 Kv.

RMS Magnet Current --- 136 Amperes.

Injection Pulse Width --- Approximately 7 microseconds at base.

Average Injection Current --- 100 Microamperes.

Gun Type --- General Electric, Lanthanum Boride Cathode.

Injection Timing, R.F. Start, Compensation Coils --- Adjusted for
maximum machine output as indicated with ion chamber.

Coil in Use --- B

Output Indication --- 0.55 on arbitrary scale¹.

The optimum operating condition for the 6AS7 cathode follower circuit is with the grid bias adjusted for about 30 milliamperes total direct current with no driving pulse. With the machine adjusted as above, the presence or absence of this direct current in a perturbing coil had no effect.

If a pulse width of 1.2 microseconds is used, the maximum effect on the gamma output is obtained when the perturbing pulse is of maximum amplitude and is positioned at the peak value of the injection voltage. The maximum drop in amplitude is about 20 per cent. A definite indication is obtained for this situation. If the injection

¹Pailey, Shady Olus. Constructing and testing an ionization chamber for monitoring a synchrotron beam. Unpublished M. S. Thesis. Ames, Iowa, Iowa State College Library. 1950.

pulse is either advanced or retarded to give an output less than the maximum value, a distinct indication can not be obtained, the output becoming too unstable to say decisively that an acceptance time can be discerned on either the leading or the trailing edge of the injection pulse.

If the output is maximized by the use of the correct injection timing and then decreased slightly by altering one of the compensation coil currents, an indication at the peak of the injection pulse is again obtained.

Increasing the injection voltage slightly while holding the average current to a constant value also did not alter the response of the machine. The injection timing and compensation coils were optimized at each point.

It was attempted to note the position of the acceptance period for different values of injection current while holding the injection voltage at a constant value. No change was noted for values of average injection current between 88 and 100 microamperes. Below this value of current the output of the machine became too low to allow adequate measurements to be made.

Substitution of coil C also had no effect on any measurements.

It was not feasible to measure time from an absolute time zero, as previously explained, but time can be measured from an arbitrary point someplace prior to the injection pulse.

Long perturbing pulses were first used and relative output as a function of perturbing pulse position plotted for each pulse width. Time was measured from the arbitrary point to the leading edge of the perturbing pulse. The region of negative slope for each curve was compared with that obtained for a slightly narrower perturbing pulse and if the region of negative slope became smaller with decreasing pulse width it was concluded that the acceptance time was smaller than the width of the perturbing pulse. This situation is shown in Fig. 14 where the results for a 1.2 microsecond and a 3 microsecond pulse are shown. The results indicated for the 1.2 microsecond pulse are also important because a pulse of 1 microsecond gave the same dip in output while a 1.5 microsecond pulse gave a slightly larger dip.

A pulse of 10 amperes in magnitude gave an acceptance time of approximately 1.3 microseconds while a pulse of 25 amperes indicated a slightly larger value of 1.45 microseconds. The increased acceptance time given by the larger pulse is very likely due in part to the fact that the pulses are not sufficiently rectangular in shape.

From these data it is concluded that the acceptance time is somewhere in the range of 1.4 to 1.5 microseconds in duration.

It was originally felt that perhaps increases in output with increases in injection voltage¹ might be explained by larger

¹Westendorp, W. F. and Elder, F. R. Electron gun emission and x-ray output for three betatrons. Phys. Rev. 76: 445. 1949.

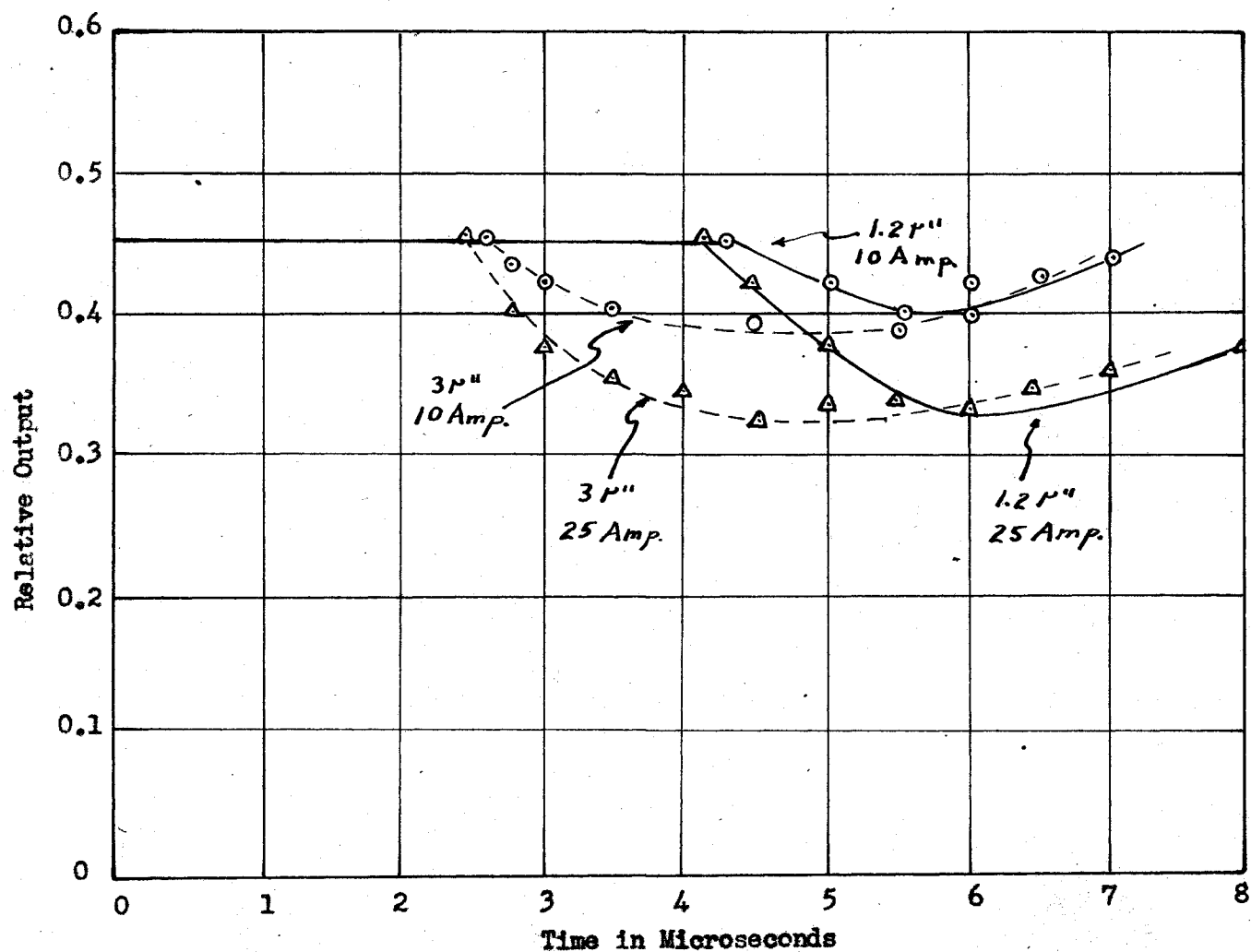


Fig. 14 Output As A Function of Perturbing Pulse Time

values of the acceptance time. However, the calculations of part IV. seem to preclude this possibility, at least for this particular machine. This is also borne out experimentally, operating the machine at an injection voltage of 52.5 Kv. did not alter the data shown in Fig. 14.

VI. SUMMARY

The original purpose of this thesis was to determine the effect of injection over a period of time which is large compared to the interval over which electrons can be accepted. This has been done for the case of constant-momentum injection; Eq. (65) shows the dependence of this acceptance period upon the momentum of injection, the radius of injection, and a complicated function of the field configuration. A relationship similar to Eq. (65) has been used to approximate acceptance periods for the ISC synchrotron, these results being given in Fig. 6.

An approximation for the case of pulsed injection has also been made. These results in general are not too satisfactory due to the extreme nature of the assumed waveform. However the results are sufficient to show some of the possibilities of momentum-matching.

There is considerable work yet to do with respect to the experimental measurement of acceptance time. The method described here has several weaknesses, one of which could be overcome by the use of a better-shaped current pulse.

In order to establish more firmly the operating characteristics of the ISC synchrotron, considerably more data should be obtained. In particular an attempt should be made to clarify the situation

existing when capture is attempted on the leading or the trailing edge of the injection pulse. Further experiments should also include injection at different values of magnet current and with a selection of injection devices.

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VIII. ACKNOWLEDGEMENTS

The writer wishes to express his appreciation to Dr. D. J. Zaffarano for his interest and advice, and to Prof. R. L. Doty for many excellent suggestions and many hours of assistance.